

Natural Gas Storage Valuation and Optimization: A Real Options Application

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Abstract

In this paper we present an algorithm for the valuation and optimal operation of natural gas storage facilities. Real options theory is used to derive nonlinear partial-integro-differential equations (PIDEs) for the valuation and optimal operating strategies. The equations are designed to incorporate a wide class of spot price models that can exhibit the same time-dependent, mean-reverting dynamics and price spikes as those observed in most energy markets. Particular attention is paid to the operational characteristics of real storage units, these characteristics include: working gas capacities, variable deliverability and injection rates and cycling limitations. We illustrate the model with a numerical example of a salt cavern storage facility that clearly shows how a gas storage facility is like a financial straddle with both put and call properties. Depending on the amount of gas in storage the relative influence of the put and call components vary.

1 Introduction

According to the U.S. Department of Energy [of Energy, 2002] the natural gas market in the United States was a \$90 billion per year industry in 1995, and currently accounts for one-fifth of the nation's energy supply. Additionally, gas-fired electrical power generation is projected to grow to an estimated 360,000 megawatts by 2004 up from 140,000 megawatts in 1994, accounting for approximately 40% of the total annual electricity generating capacity of the U.S. [Hopper, 2002]. Efficient and reliable natural gas storage is vital for managing fluctuations in gas supply and demand and provides the only significant supply and demand cushion. Traditionally, these fluctuations were the result of increased gas consumption used for heating purposes during the winter months. However, with the increased use of gas-fired electrical generators, peak summer power demand will increasingly contribute to gas demand fluctuations. The need for additional investment in gas storage facilities and technologies, and the optimal utilization of this infrastructure has never been greater. In particular, investment analysis and optimization methodologies capable of accurately accounting for the various operating characteristics of real storage facilities, and that are able to deal with the complex stochastic nature of natural gas prices are required.

There are four main operating characteristics that distinguish natural gas storage facilities. They are: base and working gas capacities, deliverability, injection capacity, and cycling. The base gas or cushion is the amount of gas required to maintain adequate reservoir pressure and is generally never removed, while working gas capacity is the amount of gas that is available to produce and sell. Deliverability refers to the rate at which the reservoir can release gas from its reserves. Deliverability is highest when the reservoir is closest to its maximum capacity and lowest when the reservoir is nearly empty. The injection capacity on the other hand refers to the rate at which natural gas can be pumped into storage for later use. In contrast to the deliverability, the injection rate is at its lowest when the reservoir is at maximum capacity and is at its highest when the reservoir is empty. Cycling refers to the number of times that the working gas volumes can be injected and withdrawn in a year. Most of the current working gas storage facilities in the U.S.

are single cycle and are typically refilled during the months from April through October and withdrawn during the heating season November through March. Because of the low cycling rate, single cycle storage facilities cannot serve summer gas-fired power demand while still meeting winter demand requirements. High deliverability multiple cycle (HDMC) facilities on the other hand can cycle several times per year and are thus the only facilities capable of servicing the growing gas-fired electrical power market. The differences in operating characteristics depends on the nature of the gas storage facility in question.

There are three main types of natural gas storage facilities widely in use in the U.S. and Canada: depleted oil and gas reservoirs, salt cavern storage, and aquifer storage. Each of these facilities has its own advantages and disadvantages. Depleted reservoirs tend to have the lowest deliverability and injection rates, and are typically single cycle facilities which cycle just once per year. They also typically require very large amounts of base gas. Despite these drawbacks, depleted reservoir storage accounts for most of the U.S. working gas storage capacity due to the fact that these facilities tend to be abundant in the high population density U.S. northeast where 56% of current U.S. storage capacity is located. Salt cavern storage facilities in contrast, tend to have very low base gas requirements, the highest deliverability and injection rates, and high cycling characteristics, sometimes as much as 4 or 5 times per year. This high deliverability multiple cycling characteristic means that such facilities are capable of daily production and nightly injection in order to meet peaking gas demands from gas-fired power plants. The major drawback with salt cavern storage is that the facilities tend to be concentrated mainly in the Gulf Coast area with the exception of southern Ontario and southern Michigan where there are 61 active salt caverns in Lambton County Ontario and 22 in St. Clair County Michigan [Hamilton, 1995]. Aquifer storage lies somewhere in between the depleted reservoir, and salt cavern extremes. Aquifer storage facilities are located close to end user markets, and have deliverability, injectivity, and cycling characteristics that fall in between those of depleted reservoir and salt cavern facilities. The major disadvantage with aquifer storage is the extremely high base gas requirements often as high as 80%, most of which is unrecoverable. The accurate valuation and optimization of a gas

storage facility depends greatly on its operating characteristics. Another important consideration is the random nature of natural gas spot prices.

The role of storage facilities in the natural gas market is to balance the seasonal and intra-seasonal demand swings of gas end-users. By purchasing and injecting gas into storage during the non-heating months and releasing gas reserves during the heating months, gas storage facilities help to mitigate seasonal demand and price fluctuations. With the increase in gas-fired power generation, intra-day gas demand fluctuations caused by daily peak electricity demand will also have to be balanced by HDMC storage units. The market force of gas storage to mitigate demand fluctuations is not perfect, and predictable seasonal price trends can still be observed. Sometimes these market imperfections can lead to dramatically high and low price spikes far outside normal seasonal equilibrium levels. These price spikes are the markets way of signalling a need for more storage capacity investment. Price spikes are therefore crucial to storage valuation and control in that they represent enormous arbitrage opportunities. Making the most of these opportunities increases the value of existing storage units and in the long run contributes to a more efficient market. Gas storage investment and control decisions must be made in the face of uncertainty and the exciting new field of real options theory provides a framework for making such decisions.

The theory of real options is based on the realization that many business decisions have properties similar to those of many derivative contracts used in financial markets. For example, ignoring operating characteristics a natural gas well can be thought of as a series of call options on the price of natural gas, where the strike or exercise price is the total operating and opportunity costs of producing gas. A gas storage facility (again ignoring operating characteristics) can be thought of as a series of call and put options of different strikes. In markets where there exists a liquid secondary derivatives market, the picture is enhanced even further. Derivative prices can be used to determine the market's risk preferences and view on the probability distribution of future prices. By operating a gas storage facility in the way that maximizes the expected cash flow with respect to the market's view of future uncertainties and its risk tolerances for

those uncertainties, one can subsequently maximize the market value of the facility itself. The difficulty arises when operating characteristics and extreme price fluctuations are included. The exotic nature of real gas storage facilities and gas prices requires the development of new methodologies both from the theoretical as well as the numerical perspective.

The operating characteristics of a real storage facility pose a theoretical challenge due to the nature of the opportunity cost structure. When gas is released from storage the opportunity to release that gas in the future is forgone. As well, when gas is released the deliverability of the remaining gas in storage is decreased. Similarly when gas is injected into storage both the amount and the rate of future gas injections are decreased. The opportunity costs (and thus the exercise price) varies nonlinearly with the amount of gas in the reservoir. These facts, coupled with the complicated nature of gas prices have serious implications for numerical valuation and control.

There are three common numerical techniques used in option pricing: Monte-Carlo simulation, binomial/trinomial trees, and numerical partial differential equation (PDE) techniques. Monte-Carlo simulation is flexible in terms of being able to handle a wide range of underlying uncertainties but it cannot handle problems for which an optimal exercise strategy needs to be determined especially when that strategy may be complicated. Binomial and trinomial trees can handle problems which require an optimal exercise strategy but, not in the case of natural gas storage. There are two reasons for this. Firstly such trees are just explicit finite difference methods for solving parabolic PDEs [Hull, 1999] . This paper demonstrates that in the case of natural gas storage the operating characteristics lead to equations of a parabolic and hyperbolic nature. Hyperbolic equations require far more sophisticated techniques than can be achieved with trees. Secondly, tree procedures are local in nature and as such they are limited in the types of underlying stochastic processes they can handle. In particular, trees cannot handle price spikes which are non-local; these are best modelled by Poisson processes. As price spikes are one of the market's ways of signalling a need for more gas storage investment, models which cannot include such behavior may have a serious flaw. This leaves but

one alternative, numerical PDE solvers, or more precisely (as we shall illustrate) numerical techniques for solving non-linear partial-integro differential equations (PIDEs).

In this paper we extend current real options theory and numerical technology for use in the valuation and optimal operation of natural gas storage facilities. In particular the proposed methodology is designed to accurately incorporate the various operating characteristics of real storage facilities, and is capable of dealing with the complex stochastic nature of natural gas prices. We begin by deriving a class of non-linear PIDEs the solution of which will simultaneously determine the expected cash flow and the optimal operating strategy of a general gas storage facility. We then illustrate the numerical implementation of this general theory with an example of a typical salt cavern facility. To the best of our knowledge nowhere in the academic literature has the problem of natural gas storage optimization been addressed

2 New General Gas Storage Equation

Let us begin by defining the relevant variables and parameters. Let

- \mathbf{P} = the current price per unit of natural gas.
- \mathbf{I} = the current amount of working gas inventory.
- \mathbf{c} = the control variable that represents the amount of gas currently being released from ($c > 0$) or injected into ($c < 0$) storage.
- \mathbf{I}_{\max} = the maximum storage capacity of the facility.
- $\mathbf{c}_{\max}(\mathbf{I})$ = the maximum deliverability rate, i.e. the maximum rate at which gas can be released from storage as a function of inventory levels.
- $\mathbf{c}_{\min}(\mathbf{I})$ = the maximum injection rate, i.e. the maximum rate at which gas can be injected into storage ($c_{\min}(I) < 0$) as a function of inventory levels.

- $\mathbf{a(I,c)}$ = the amount of gas that is lost given c units of gas are being released from ($c > 0$) or injected into ($c < 0$) storage and I units are currently in storage. (When gas is injected into storage fuel is needed to power the injection process, gas can also be lost due to reservoir seepage. The rate of seepage depends on inventory levels.)

With this representation, our objective is to maximize the expected cash flows of the gas storage facility under the risk adjusted measure. Since the instantaneous cash flow is simply the product of the amount of gas lost, bought or sold i.e. $(c - a(c, I))$ and the current gas price P , this objective can be written as

$$\max_{c(P,I,t)} E\left[\int_0^T e^{-\rho\tau} (c - a(I, c))P d\tau\right] \quad (1)$$

subject to

$$c_{\min}(I) \leq c \leq c_{\max}(I)$$

where $E[\cdot]$ represents expectation under the risk neutral measure over the random variable P . T denotes the time interval of consideration and ρ is the risk free interest rate. In order to solve this optimal control problem we require equations for the dynamics of I and P .

The change in I must obey the ordinary differential equation

$$dI = -(c + a(I, c))dt. \quad (2)$$

In other words the change in I is simply the negative of the sum of the controlled amount released (c) for sale and the amount lost due to seepage, pumping or any other frictional effects (a). We now require equations for the risk adjusted dynamics of the price process P .

Figure 1 depicts Henry Hub natural gas prices from 1995 through 1999 and clearly illustrates that natural gas prices can exhibit extreme price fluctuations unlike those of virtually all other commodities. Due in part to imperfections in the gas storage market the price of natural gas exhibits enormous price spikes in which prices may jump orders of magnitude in a short period of time and then return to normal levels just as

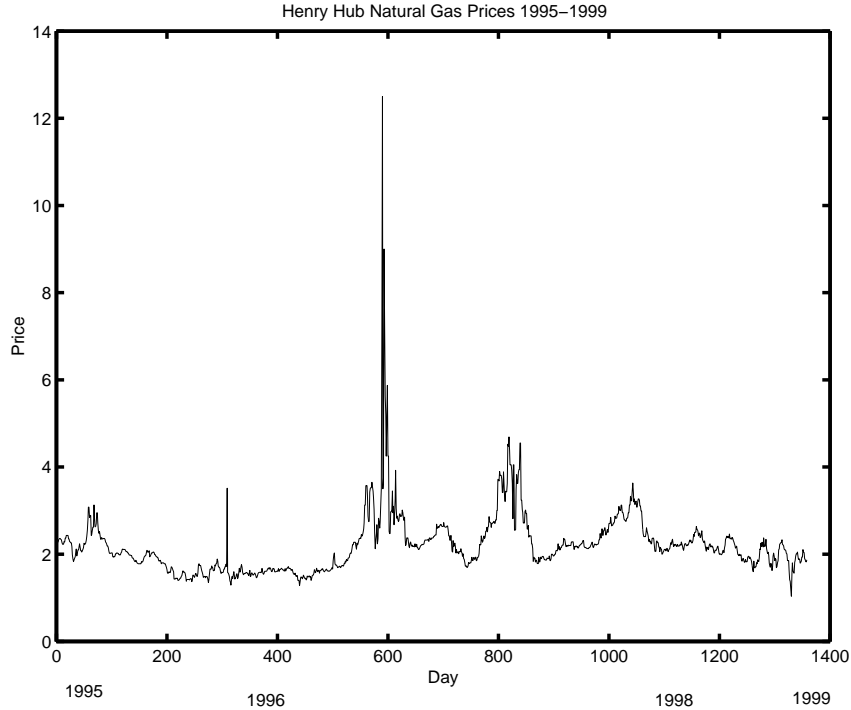


Figure 1: **Time-series of Henry Hub natural gas prices 1995-1999.**

quickly. What constitutes a normal level varies depending upon the time of year. No generally agreed upon stochastic model exists for natural gas prices (some non-price-spike models can be found in [Pilipovic, 1998]). For this reason a general valuation and control algorithm must be flexible enough to deal with a wide range of potential spot price models while remaining computationally tractable. The most general form of a continuous time Markov process for the risk adjusted price of natural gas (P) in one dimension can be written as:

$$dP = \mu_1(P, t)dt + \sigma_1(P, t)dX_1 + \sum_{k=1}^N \gamma_k(P, t, J_k)dq_k \quad (3)$$

where μ , σ , and the γ_k 's can be any arbitrary functions of price and/or time, and the J_k 's are drawn from some other arbitrary distributions $Q_k(J)$. dX_1 denotes the standard increment of Brownian motion while

the dq_k 's are Poisson processes with the properties

$$dq_k = \begin{cases} 0 & \text{with probability } 1 - \epsilon_k(P, t)dt \\ 1 & \text{with probability } \epsilon_k(P, t)dt, \end{cases}$$

In other words with probability $\epsilon_k(P, t)dt$, $dq_k = 1$, and P jumps by an amount $\gamma_k(P, t, J_k)$ to $P + \gamma_k(P, t, J_k)$ where J_k is drawn from some probability distribution $Q_k(J)$. The number of Poisson processes (N) is also arbitrary. Aside from dramatically increasing the available range of potential probability distributions, one of the advantages of adding multiple Poisson processes is that, from a computational viewpoint, using $N + 1$ such processes requires no more computational effort than N processes. The drawback of course is the difficulty in estimating the large number of parameters. This is where having a liquid secondary derivatives market can be invaluable from the point of view of parameter estimation and spot model validation.

Higher dimensional models for natural gas price dynamics which are dependent on additional random factors can be imagined. Such models may include stochastic volatility or stochastic mean reversion. One could also imagine path dependent models in which the price dynamics depend on the current total amount of storage in a given market. Adding additional random factors is conceptually a straightforward exercise and the equations we will derive can readily be extended to higher dimensions, however doing so increases the computational complexity of the problem and therefore must only be done judiciously. Typically, two or three factors of the form (3) could be used for the price model without too much computational difficulty. In this paper we will not explore such spot models any further.

Now that we have representations for the dynamic processes which govern the variables I and P , we can set out to derive equations for the corresponding optimal strategy $c(P, I, t)$ and the corresponding optimal value $V(P, I, t)$. Let

$$V(P, W, f, t; c) = \max_c E \left[\int_t^T e^{-\rho(\tau-t)} (c - a(I, c)) P d\tau \right]. \quad (4)$$

This equation can be rewritten as

$$V = \max_c E \left[\int_t^{t+dt} e^{-\rho(\tau-t)} (c - a(I, c)) P d\tau + \int_{t+dt}^T e^{-\rho(\tau-t)} (c - a(I, c)) P d\tau \right]$$

$$\begin{aligned}
&= \max_c E \left[\int_t^{t+dt} e^{-\rho(\tau-t)} (c - a(I, c)) P d\tau + e^{-\rho dt} \int_{t+dt}^T e^{-\rho(\tau-(t+dt))} (c - a(I, c)) P d\tau \right] \\
&= \max_c E \left[\int_t^{t+dt} e^{-\rho(\tau-t)} (c - a(I, c)) P d\tau + e^{-\rho dt} V(P + dP, I + dI, t + dt) \right]
\end{aligned}$$

which is simply the Bellman equation. Employing Ito's lemma we expand the above in a Taylor's series using equations (3), and (2) to yield

$$\begin{aligned}
V &= \max_c E \left[(c - a(I, c)) P dt + (1 - \rho dt) V \right. \\
&\quad + (1 - \rho dt) \left(V_t + \frac{1}{2} \sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I, c)) V_I \right) dt \\
&\quad \left. + (1 - \rho dt) \sum_{k=1}^N (V_k^+ - V) dq_k + (1 - \rho dt) (\sigma_1 V_P dX_1) \right]
\end{aligned}$$

where $V^+ = V(P + \gamma_k(P, t, J_k), I, t)$ denotes the value of V given that jump process k has occurred.

Eliminating all terms that go to zero faster than dt and simplifying shows that

$$\begin{aligned}
0 &= \max_c E \left[\left(V_t + \frac{1}{2} \sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I, c)) V_I + (c - a(I, c)) P - \rho V \right) dt \right. \\
&\quad \left. + \sum_{k=1}^N (V_k^+ - V) dq_k + \sigma_1 V_P dX_1 \right].
\end{aligned}$$

Taking expectations and dividing through by dt gives

$$\max_c \left[V_t + \frac{1}{2} \sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I, c)) V_I - \rho V + (c - a(I, c)) P + \sum_{k=1}^N \epsilon_k E[V_k^+ - V] \right] = 0$$

where the remaining expectation is taken with respect to the size of the jump J_k drawn from the probability density functions $Q_k(J)$. Only two terms in the above equation involve c , so the optimal value for c maximizes

$$\max_c [-(c + a(I, c)) V_I + (c - a(I, c)) P] \tag{5}$$

subject to

$$c_{\min}(I) \leq c \leq c_{\max}(I).$$

This result implies that when $c(P, I, t)$ is chosen to maximize equation (5) then

$$V_t + \frac{1}{2} \sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I, c)) V_I - \rho V + (c - a(I, c)) P + \sum_{k=1}^N \epsilon_k E[V_k^+ - V] = 0. \tag{6}$$

We now have two conditions which will allow us to simultaneously determine the expected cash flows $V(P, I, t)$ and the optimal strategy $c(P, I, t)$. It remains only to define boundary conditions.

The terminal condition at time T is evident from equation (4), and is given by

$$V(P, I, T; c) = 0. \tag{7}$$

Two boundary conditions are required in P . The appropriate boundary conditions depend on the specific spot price model (3) chosen, however in most cases the boundary conditions

$$\begin{aligned} V_{PP} &\rightarrow 0 \quad \text{for } P \text{ large} \\ V_{PP} &\rightarrow 0 \quad \text{as } P \rightarrow 0 \end{aligned}$$

are sufficient. The physical interpretation of these boundary conditions is that as P gets very large/small the optimal strategy is to release/pump the gas as fast as possible, and a small change in P will not alter that strategy. This condition implies that V must vary linearly with respect to P in these regions.

Equation (6) is hyperbolic in I as it depends only on the first derivative with respect to I . As such only one boundary condition is ever relevant at every given point in space and time. However, the relevant boundary condition depends on the direction of gas flow. If gas is flowing into the reservoir and I is increasing then the relevant boundary information is that one cannot fill the reservoir beyond its maximum capacity. Thus boundary information flows into the solution domain from larger values of I . In other words when approximating derivatives in I with a finite difference grid, if $(c + a(I, c)) < 0$ reservoir levels are increasing and one must only use information at values of I greater than or equal to the current level. If $(c + a(I, c)) > 0$ then gas is flowing out of the reservoir and I is decreasing in which case the relevant boundary information is that one cannot release more working gas than one has. Thus boundary information flows into the solution domain from smaller values of I and derivative approximations must only use information at values of I less than or equal to the current level. This concept is referred to as upwind differencing in the computational

fluid dynamics literature. The resulting boundary conditions are therefore

$$\begin{aligned} (c + a(I, c)) &\geq 0 \quad \text{for } I = I_{\max} \\ (c + a(I, c)) &\leq 0 \quad \text{for } I = 0. \end{aligned}$$

Thus when $I = I_{\max}$ gas cannot be pumped into the reservoir, boundary information flows from below and all necessary derivatives must be calculated from values of I less than or equal to I_{\max} . The reverse is true for $I = 0$.

The hyperbolic nature of equation (6) has other consequences as well. Due to the fact that there is no diffusion in the I dimension second order accurate upwind differencing schemes can suffer instabilities and wild spurious numerical oscillations can occur. One approach is to use a first order upwind differencing scheme. The drawback with this approach is that it is very inaccurate and spatial step sizes need to be very small. This can drastically and needlessly increase the computational complexity of the problem. The reason why first order upwind differencing schemes work on such hyperbolic problems is that they actually create an artificial numerical diffusion. In essence they work because they are actually solving a parabolic problem. This insight lead researchers to a class of numerical schemes known as total variation diminishing (TVD) schemes. The basic premise of such techniques is to use a second order accurate scheme everywhere except in places in which the slope is changing rapidly, when this occurs a tiny amount of artificial diffusion is introduced to prevent instability. In this way one can achieve stability and preserve second order accuracy see [LeVeque, 1992]. A full discussion is beyond the scope of this paper and interested readers are referred to [LeVeque, 1992]. Our experience trying a variety of TVD methods indicates that the minmod slope limiting method works best for the gas storage problem.

TVD methods work exceptionally well at achieving high accuracy and stability. However, they only prevent instabilities in the solution V itself. Since the control schedule depends on V_I it is possible that the control could be off in places for which the derivative is changing very quickly. The solution to this problem however is simple: derive an equation for V_I and use the TVD method to find the solution to this problem

and thus the control schedule. Then simply substitute the control schedule directly into the equation for V and solve. If we let $W = V_I$, and differentiate equation (6) with respect to I we obtain the equation

$$\max_c \left[W_t + \frac{1}{2} \sigma_1^2 W_{PP} + \mu_1 W_P - (c + a(I, c)) W_I - a_I(I, c) W - \rho W + -a_I(I, c) P + \sum_{k=1}^N \epsilon_k E[W_k^+ - W] \right] = 0.$$

The best way in which to illustrate the preceding theory is with the use of a specific example.

3 Realistic Example Problem

Example:

The Texas based Stratton Ridge salt cavern gas storage facility has a working gas capacity of 2000 MMcf. At maximum capacity the facility has a max deliverability of 250 MMcf/day and a maximum injectivity (which occurs at minimum capacity), of 80 MMcf/day [Dietert and Pursell, 2000]. Assume that the facility has a base gas requirement of 500 MMcf and is available for lease for a one year duration. The base gas is already built into the facility and cannot be removed. Assume that the injection pump requires a constant input of 1.7 MMcf of natural gas per day (which is typical). Furthermore, assume for simplicity of exposition that the ideal gas law and Bernoulli's equation apply and that the risk adjusted spot price dynamics of natural gas are given by the following mean reverting jump diffusive process:

$$dP = .25 * (2.5 - P)dt + .2PdX + (J - P)dq. \quad (8)$$

Where

$$dq = \begin{cases} 0 & \text{with probability } 1 - 2dt \\ 1 & \text{with probability } 2dt, \end{cases}$$

and J is normally distributed with mean 6 and variance 4 i.e.

$$J \in N(6, 4).$$

Prices are quoted in MMBtus which are roughly equivalent to one thousand cubic feet, and time is assumed to be measured in years. The facility has no detectable gas leakage and the risk free interest rate is 10% per year.

Solution:

Since there is no detectable gas leakage and, since the injection pump operates at a fixed input of 1.7 MMcf/day the function \mathbf{a} of the previous section is given by:

$$a(I, c) = a(c) = \begin{cases} 0 & \text{for } c \geq 0 \\ 1.7 * 365 & \text{for } c < 0 \end{cases} \quad (9)$$

as measured on an annualized basis. With this result in hand we can substitute the spot price model (8) into equation (6) to obtain

$$V_t + \frac{1}{2}.04P^2V_{PP} + .25(2.5 - P)V_P - (c + a(c))V_I - .1V + 1000(c - a(c))P + 2 \int_0^\infty (V(J, I, t) - V(P, I, t)) \frac{1}{\sqrt{8\pi}} e^{-(J-6)^2/(8)} dJ = 0. \quad (10)$$

The optimal value for c is the solution to the optimization problem

$$\max_{c_{min}(I) \leq c \leq c_{max}(I)} [-(c + a(c))V_I + (c - a(c))1000P].$$

From equation (9) it is easy to see that

$$c = \begin{cases} c_{max}(I) & \text{for } 1000P > V_I \\ c_{min}(I) & \text{for } (c_{min}(I) - 1.7 * 365)1000P < (c_{min}(I) + 1.7 * 365)V_I \\ 0 & \text{otherwise.} \end{cases}$$

All that remains is to determine $c_{min}(I)$ and $c_{max}(I)$.

Let P_{in} be the pressure of the gas inside the storage container and I_b denote the amount of base gas, then the ideal gas approximation with fixed temperature and volume states that the pressure inside the storage vessel is proportional to the total amount of gas in the storage vessel, i.e.

$$P_{in} = \alpha(I + I_b)$$

for some constant of proportionality α . If P_{out} , ρ_{out} , and v_{out} represent the pressure, density, and velocity of the gas exiting the storage container respectively, then Bernoulli's law states that

$$\begin{aligned} P_{out} + \frac{1}{2}\rho_{out}v_{out}^2 &= P_{in} \\ &= \alpha(I + I_b). \end{aligned}$$

The pressure, P_{out} of the gas outside the storage tank must be equal to the pressure inside the tank when only the base gas is present i.e.

$$P_{out} = \alpha I_b.$$

This implies that

$$v_{out} = \sqrt{\frac{2\alpha}{\rho_{out}}I}.$$

Thus the relationship between the velocity of the gas as it leaves the storage vessel and the amount of working gas in storage has been determined. The deliverability is the flow rate of the gas that leaves storage in a day, and as such the maximum deliverability is the product of the velocity and the maximum surface area of the opening from the storage container into the pipeline. Therefore we deduce that the maximum flow rate $c_{max}(I)$ must be proportional to the square root of the amount of working gas in storage i.e.

$$c_{max}(I) = K\sqrt{I}$$

for some constant K . We know that the maximum deliverability is 250 MMcf/day and occurs at maximum capacity $I = 2000$ MMcf, therefore if time is measured in years we must have

$$250 * 365 = K * \sqrt{2000}$$

hence

$$c_{max}(I) \approx 2040.41\sqrt{I}. \tag{11}$$

Turning our attention to the injection rate, the ideal gas law once again states

$$P_{in} = \alpha(I + I_b).$$

Since the storage tank has a fixed volume it must be that the density of the gas in containment is also proportional to the total amount of gas in the reservoir i.e.

$$\rho_{in} = \beta(I + I_b)$$

for some constant β .

Assuming the pump produces a constant pressure head P_{out} , Bernoulli's law states that

$$\begin{aligned} P_{out} &= P_{in} + \frac{1}{2}\rho_{in}v_{in}^2 \\ &= \alpha(I + I_b) + \frac{1}{2}\beta(I + I_b)v_{in}^2 \end{aligned}$$

where v_{in} is the velocity of the gas being injected into the storage reservoir. In other words

$$v_{in} = \sqrt{2(P_{out} - \alpha(I + I_b))/(\beta(I + I_b))}.$$

The maximum injection flow rate is just the velocity of the injecting gas times the max surface area of the injection pipe. This results in the following expression for the relationship between the max injection rate $c_{min}(I)$ and the amount of working gas I :

$$c_{min}(I) = -K_1\sqrt{\frac{1}{I + I_b} + K_2}$$

for some constants K_1 and K_2 as yet to be determined.

When the storage reservoir is at maximum capacity the pump can no longer inject more gas, hence $c_{min}(2000) = 0$. The base gas requirement was said to be 500 MMcf and so

$$0 = -K_1\sqrt{\frac{1}{2000 + 500} + K_2}$$

which means that

$$K_2 = \frac{-1}{2500}.$$

We also know that the maximum injectivity occurs at $I = 0$ and is given by 80 MMcf/day, in annual terms this implies that

$$80 * 365 = K_1\sqrt{\frac{1}{500} - 1/2500}$$

which means

$$K_1 = 730,000.$$

Hence we conclude that

$$c_{min}(I) = -730000\sqrt{\frac{1}{I+500} - \frac{1}{2500}} \quad (12)$$

as measured on an annualized basis.

Using a simple explicit finite difference scheme, we solved the nonlinear PIDE problem (10). The results of the calculations are shown in Figures 2, 3 and 4. The trapezoid rule was used for calculating the integrals and a second order accurate upwind minmod slope limiter was employed to evaluate the derivative with respect to the hyperbolic variable I . Details of the slope limiter can be found in [LeVeque, 1992]. The hyperbolic nature of equation (10) can cause simple second order accurate upwind differencing schemes to exhibit spurious numerical oscillations. The slope limiter avoids these complications. This explicit scheme is only first order accurate in the time dimension and many faster and more stable algorithms exist to improve performance. The advantage of using a simple explicit technique is that the program can be written quickly and easily.

Figure 2 depicts the control strategy for operating the gas storage facility at the beginning of the lease with one year remaining in the contract. Three regions are clearly visible: the negative region which corresponds to pumping gas into the reservoir, the flat zero region which corresponds to doing nothing, and the high positive region corresponding to the release of gas from the reservoir. For low gas inventory levels, gas can be pumped into the reservoir very quickly (high injectivity) this can be seen from the fact that the region corresponding to negative values of c is deeper (more negative) for low gas inventory levels than for high inventory levels. In contrast when inventory levels are low, gas that is released from storage has low deliverability hence the region corresponding to positive values of c is lower for low gas inventory levels and higher for high inventory levels. The zero region of the control strategy surface is thinner for low levels of inventory and thicker for high levels of inventory. This is because it is cheaper per unit of gas to inject into

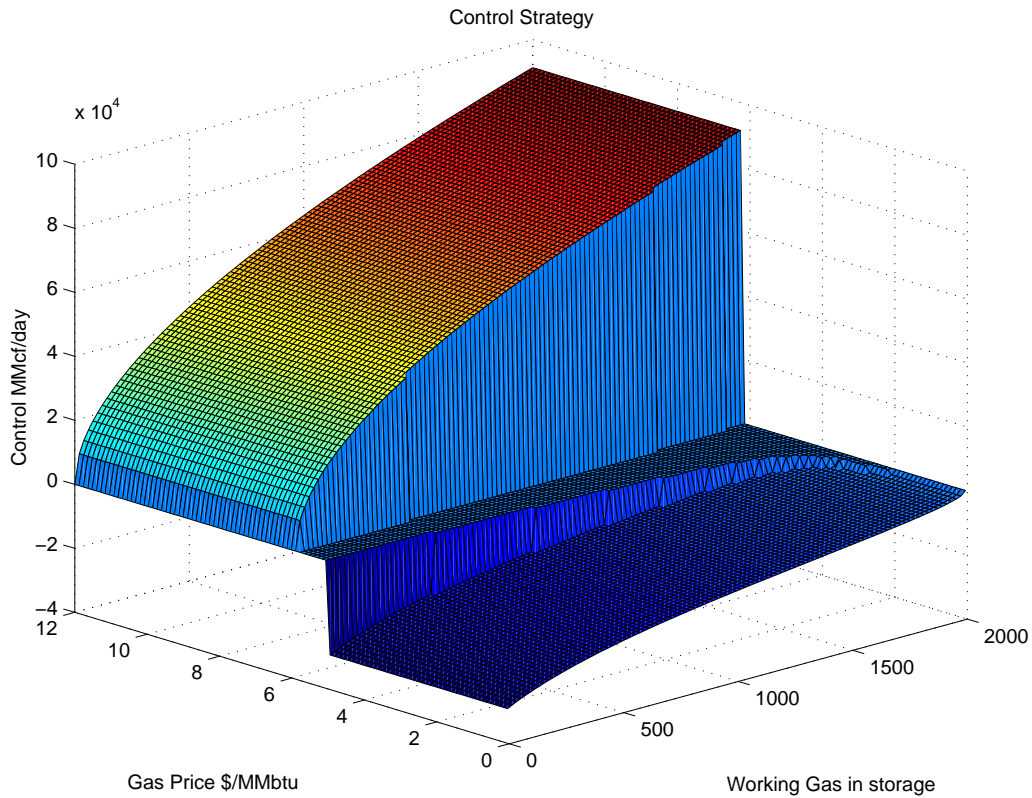


Figure 2: **Control strategy surface**

an empty reservoir than it is to inject into a reservoir that is close to maximum capacity.

Figure (3) shows the value surface of the lease. It ranges in price from a low of 1 million dollars when prices are high and the reservoir is empty, to a high of nearly 25 million dollars when prices are high and the reservoir is full. This result is very intuitive. If prices are extremely high and we have a reservoir full of gas that we can sell it is the best possible scenario. On the other hand if the reservoir is empty there is nothing to sell, and if prices are high its not worth it to inject any gas into storage and so the reservoir will continue to remain empty in the near future.

Figure (4) shows cross sectional slices of Figure (3) along various slices of the inventory axis. We see from the first graph that when there is no gas in the reservoir, the value of the gas storage unit is higher the lower

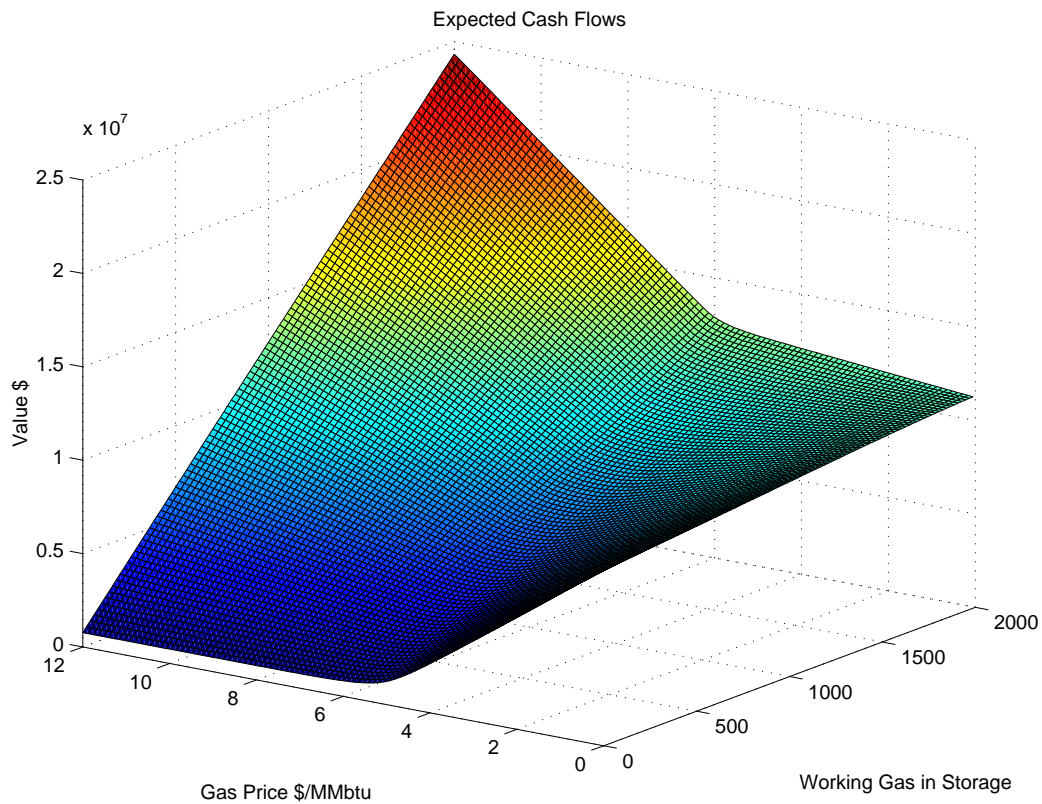


Figure 3: **Value surface**

the price of gas. This property is due to the fact that if there is no gas in reserve none can be released, hence the gas storage unit is in this case, essentially just a put option on gas prices, and as such it is more valuable when prices are low. In contrast, the fourth graph of this Figure, depicts the value of the facility when the working gas reserves are at maximum capacity. In this case the value is highest for high prices and lowest for low prices. This property is reasonable since at high reservoir levels the gas storage unit is essentially just a call option on gas prices. For inventory levels somewhere between maximum and minimum capacity the gas storage facility is like a financial straddle with both put and call properties. Graphs 2 and 3 of Figure (4) illustrate this fact. Both figures show an increase in value for very low gas prices as well as for very high gas prices, as in either case income can be generated. The second graph corresponds to $I = 200$ MMcf or

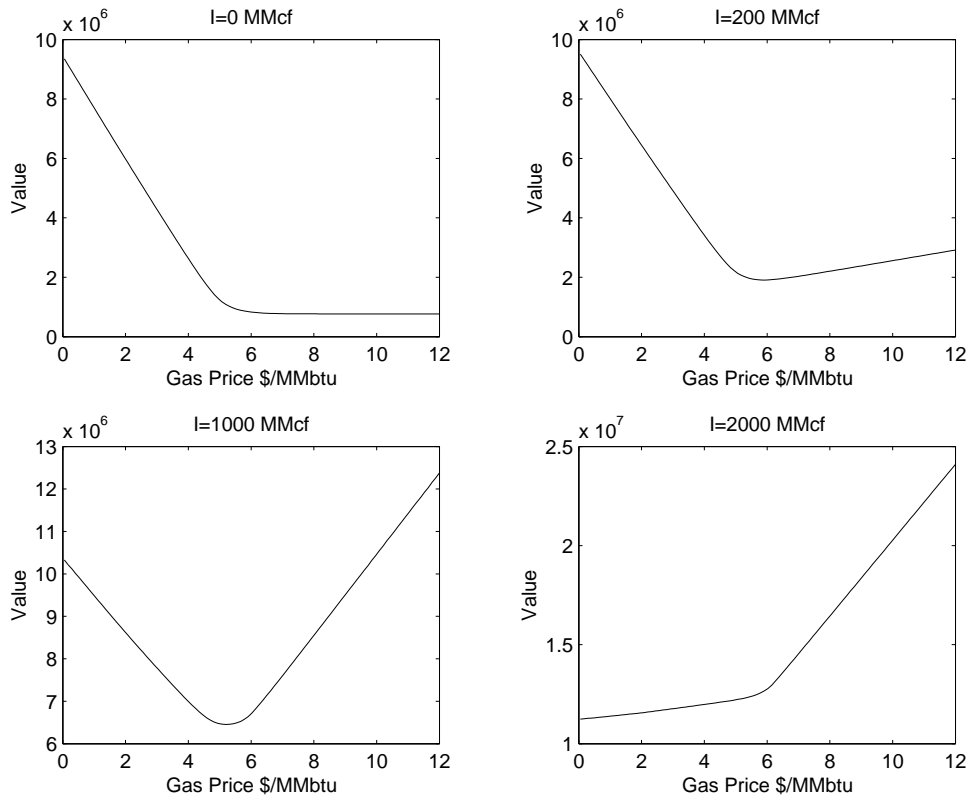


Figure 4: **Cross-sectional slices of the value surface along the I axis**

reserve levels at only 10% capacity, in this case the reservoir is still a straddle but is closer to being a put and hence more heavily weighted for low gas prices. The third plot in the series corresponds to inventory at half capacity and clearly illustrates straddle-like characteristics.

4 Conclusions

In this paper we derived models for the valuation and optimal operation of natural gas storage facilities that incorporates realistic price dynamics and operational characteristics. The approach presented has several advantages: it is able to directly account for price spikes, and can attain second order accuracy in terms of

operational state specification. It has the flexibility of Monte-Carlo and based techniques and the speed and accuracy of trinomial tree approaches. And although the specific implementation presented in this paper was only first order accurate in time, it can be made second order accurate in the time dimension.

Aside from determining the optimal operating strategy, helping to forecast expected cash flows, and determining the value of leasing agreements the framework developed here may also be used to evaluate the economic trade-offs involved in building new facilities. For example, what is the relative importance of deliverability and injectivity versus working capacity in determining the value? Given the current price dynamics and relative costs what type of storage unit is most appropriate. in a given market? When designing a pump storage facility what dimensions optimal given their respective costs?

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